Normal Subgroups

These are due Tuesday, May 8.

If G is a group and K a subgroup, recall from last week that K is *normal* in G if $gKg^{-1} = K$ for all $g \in G$. This means that if $k \in K$ then $gkg^{-1} \in K$.

1. Let $G = S_3$ and $K = \{1, (123), (132)\}$, $H = \{1, (12)\}$. Explain why K is normal and H is not.

2. Find a normal subgroup V of S_4 of order 4. (This means it has 4 elements.)

If G is a group and H a subgroup, for any $x \in G$, the set $xH = \{xh | h \in H\}$ is a *left coset*. The set $Hx = \{hx | x \in G\}$ is a *right coset*.

3. Show that if xH and yH are left cosets then either $xH = yH$ or $xH \cap yH = \varnothing$. Thus two cosets are either equal or disjoint. (**Hint:** if $z \in xH \cap yH$ show that $xH = zH = yH$.)

4. Show that if $G = S_3$ and $H = (12)$ then with $x = (13)$ the cosets xH and Hx are not the same. (Compute the two cosets.) But show that $xK = Kx$.

5. Let G be a group and K a normal subgroup. If xK and yK are cosets, prove that $xK \cdot yK = xyK$. (**Hint:** Use normality!) Deduce that the set G/K of cosets of K form a subgroup of G, called the *quotient group*.