Math 210C Take-Home Midterm

Due Friday, May 1 before 5 PM.

1. Show that the group with generators and relations

$$G = \langle x, y | x^{11} = y^5 = 1, yxy^{-1} = x^4 \rangle$$

has order 55. Find the character table by finding the conjugacy classes and irreducible characters. Construct the non-linear characters as induced from an appropriate subgroup.

2. Let G be the group of order 27:

$$G = \left\{ \begin{pmatrix} 1 & x & y \\ & 1 & z \\ & & 1 \end{pmatrix} | x, y, z \in \mathbb{F}_3 \right\}.$$

Find the character table by finding the conjugacy classes and irreducible characters. Construct the non-linear characters as induced from an appropriate subgroup.

3. Let G be a group acting on a set X. Assume that the action is transitive. Let H be the isotropy subgroup of some $x_0 \in X$. Let χ be the permutation character of G determined by this group action, namely

$$\chi(g) = \#\{x \in G | gx = x\}.$$

Show that if 1 denotes the trivial character of H then the induced character 1^G is χ.