

Handout 3: Conjugation Worksheet

Here are short easy questions about conjugation, for Math 88Q. The text for these is `rubik.pdf` Chapter 5. You can read this chapter without reading everything that comes before it. The purpose of these problems is to establish certain points that you *must* understand for us to talk about conjugation. Some of them are really answered directly in the notes or in class. Due Tuesday April 24.

1. In the symmetric group S_4 compute ghg^{-1} for the following examples. I've done the first one:

$$(124)(123)(124)^{-1} = (124)(123)(142) = (243)$$

g	h	ghg^{-1}
(124)	(123)	(243)
(12)	(123)	
(1234)	(123)	
(12)(34)	(123)	
(124)	(13)(24)	
(12)	(13)(24)	
(1234)	(13)(24)	
(12)(34)	(13)(24)	

- Observe that h and ghg^{-1} have the same “shape.” For example when $h = (123)$ we find that ghg^{-1} is a 3-cycle for all g . Explain why this is always true.
- What is the definition of an “equivalence relation?” Specifically, let X be a set and \sim is a relation on X , so that if $x, y \in X$ then $x \sim y$ is either true or false. What conditions does \sim have to satisfy to be called an equivalence relation?
- “If \sim is an equivalence relation on X then X is partitioned into disjoint equivalence classes.” Explain.
- If $x, y \in G$ (a group) define $x \sim y$ to mean that $x = gyg^{-1}$ for some $g \in G$. Prove that this is an equivalence relation on G , called *conjugation*. In view of problem 2, the conjugacy classes in the symmetric group S_4 each consists of permutations with a given “shape.” For example, the set of all three cycles is a conjugacy class:

$$\{(123), (132), (124), (142), (134), (143), (234), (243)\}$$

Make a list of all five conjugacy classes of S_4 .